EE 232 Lightwave Devices
Lecture 20: Noises in Photodetectors

Reading: Yariv 10.3-10.5 or Chuang 15.1.3

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Poisson Distribution

Poisson distribution:

a given event occurring in any time interval is distributed uniformly over the interval.

The probability of n electrons arriving in a period T: is

$$p(n) = \frac{\bar{n}^{n} e^{-\bar{n}}}{n!}$$

where $\bar{n}$ is the average number of electrons arriving in T

Properties of Poisson Distribution:

Mean = $\bar{n}$

Variance = $\bar{n}$

Spectral Density Function

Random variable $i(t)$ consists of a large number of individual events (e.g., single-electron photocurrent) at random time:

$$i(t) = \sum_{i=1}^{N_T} f(t - t_i), \qquad 0 \le t \le T$$

Fourier transform: $I_T(\omega) = \sum_{i=1}^{N_T} F_i(\omega)$

$$F_i(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t - t_i)e^{-i\omega t}dt = \frac{e^{-i\omega t_i}}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt = e^{-i\omega t_i}F(\omega)$$

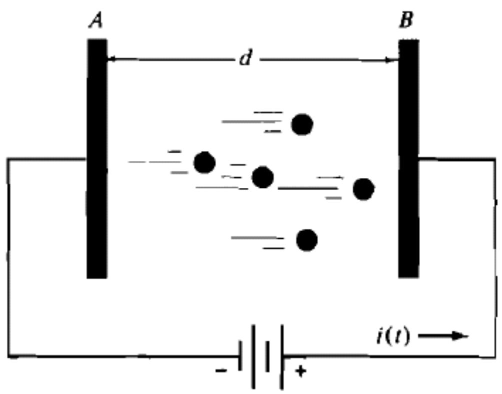
$$\left\langle \left|I_T(\omega)\right|^2 \right\rangle = \left\langle \left|F(\omega)\right|^2 \left\{ N_T + \sum_{i=1}^{N_T}\sum_{j=1}^{N_T} e^{-i\omega(t_i - t_j)} \right\} \right\rangle = \overline{N_T}\left|F(\omega)\right|^2 = \overline{N}T\left|F(\omega)\right|^2$$

$\overline{N}$: average rate of electron arrival

Spectral density function: $S(v) = \lim_{T\to\infty} \frac{8\pi^2 \left|I_T(2\pi v)\right|^2}{T} = 8\pi^2 \overline{N}\left|F(2\pi v)\right|^2$

Shot Noise

Shot Noise: Noise current arising from random generation and flow of mobile charge carriers.



Current pulse due to a single electron moving at $v(t)$:

$$i_e(t) = \frac{ev(t)}{d}$$

Fourier transform: $F(\omega)=\frac{1}{2\pi}\frac{e}{d}\int_0^{t_a} v(t)e^{-i\omega t}dt$

t_a : arrival time, $x(0)=0, \quad x(t_a)=d,$

Small transit time t_a, $\omega t_a << 1 \quad \rightarrow \quad e^{-i\omega t} \sim 1$

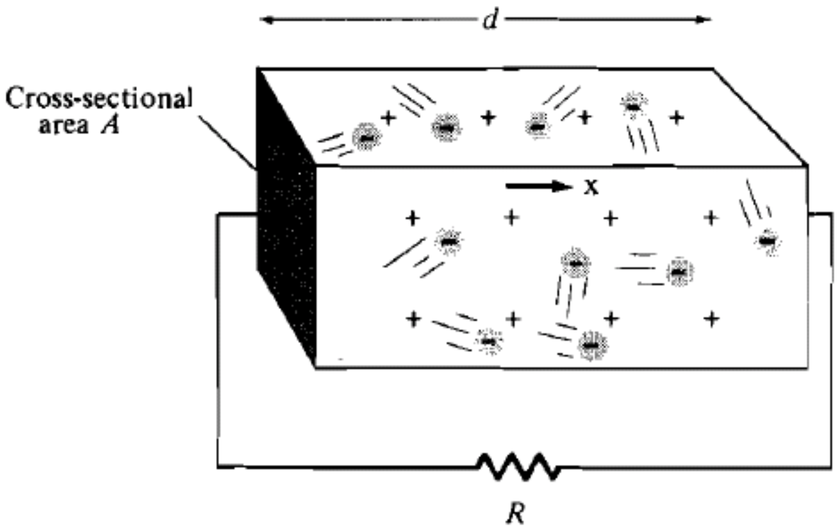
$$F(\omega)=\frac{1}{2\pi}\frac{e}{d}\int_0^{t_a}\frac{dx}{dt}\cdot 1\cdot dt = \frac{1}{2\pi}\frac{e}{d}\int_0^{d} dx = \frac{e}{2\pi}$$

$$S(v) = 8\pi^2 \overline{N}\left(\frac{e}{2\pi}\right)^2 = 2e\overline{I}$$

$$\overline{I} = e\overline{N}$$

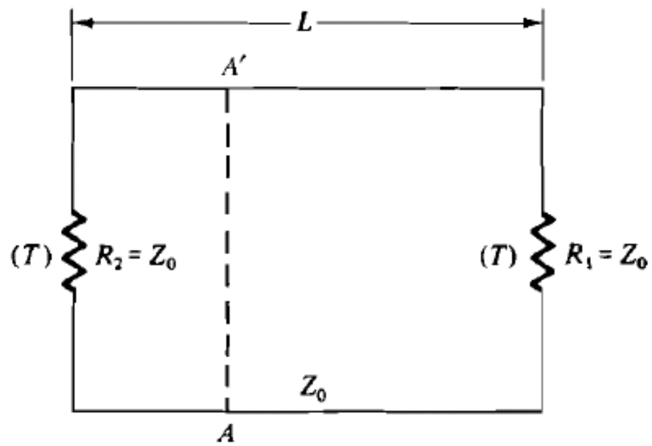
$$\boxed{\overline{i_N^2(v)} = S(v)dv = 2e\overline{I}dv}$$

Thermal Noise (Johnson Noise)



- Fluctuation in the voltage across a dissipative circuit element (resistor)
- Caused by thermal motion of charged carriers

Thermal Noise Derivation



Consider two resistors connected by a lossless transmission line of length L:

voltage wave: $v(t) = A\cos(\omega t \pm kz)$

Assume periodic condition: $kL = 2m\pi$

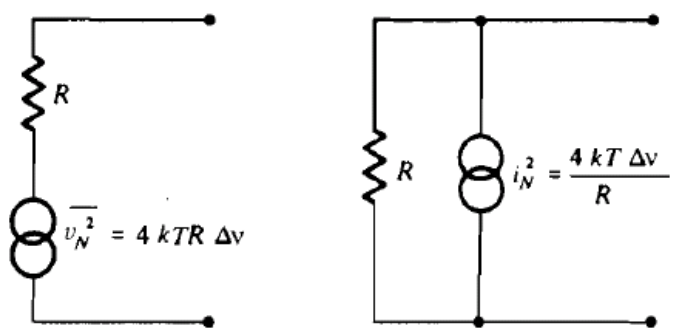
Mode density: $\rho(\nu) = \dfrac{L}{c}$

Power flow: $P = \dfrac{\text{Energy}}{\text{Transit Time}}$

$$P = \frac{1}{L/c}\left(\frac{L}{c}\Delta\nu\right)\left(\frac{h\nu}{e^{h\nu/k_BT}-1}\right) = \frac{h\nu\Delta\nu}{e^{h\nu/k_BT}-1}$$

$$h\nu / k_BT << 1$$

$$P = k_BT\Delta\nu = \left(\overline{v_N^2}\left(\frac{R}{R+R}\right)^2\right)\frac{1}{R} = \left(\overline{i_N^2}\left(\frac{R}{R+R}\right)^2\right)R$$



(a) (b)

Equivalent mean square noise voltage: $\boxed{\overline{v_N^2} = 4k_BTR\Delta\nu}$

Equivalent mean square noise current: $\boxed{\overline{i_N^2} = \dfrac{4k_BT\Delta\nu}{R}}$

Noise in p-i-n Photodiode

Noises in p-i-n photodiodes: shot noise and thermal noise

$$\overline{i_N^2(v)} = \overline{i_{N,shot}^2(v)} + \overline{i_{N,thermal}^2(v)} = 2e\bar{I}dv + \frac{4k_B T\Delta v}{R}$$

Signal:

$$i_S^2(v) = \bar{I}^2$$

Signal to noise ratio (SNR):

$$SNR = \frac{\bar{I}^2}{2e\bar{I}dv + \frac{4k_B T\Delta v}{R}}$$

Note that the SNR improves with increasing average photocurrent $\bar{I}$

Example

B = 1 GHz
R = 50Ω